Math 420/520

Assignment 1

due September 14, 2001

1. Compute the reduced row echelon form of the matrix

\[
A = \begin{bmatrix}
1 & 2 & 3 & 4 \\
5 & 6 & 7 & 8 \\
6 & 8 & 10 & 12
\end{bmatrix}.
\]

2. Let \( A \) and \( B \) be \( m \times n \) and \( n \times p \), respectively. Prove that \((AB)^T = B^T A^T\). (Show that these matrices have the same dimensions, and the \((i, j)\) entry of \((AB)^T\) is the same as the \((i, j)\) entry of \(B^T A^T\).)

3. Let \( A \in \mathbb{C}^{m \times n}\). Show that \( \text{rank}(A) = 1 \) if and only if there exist \( u \in \mathbb{C}^m \) and \( v \in \mathbb{C}^n \) such that \( A = uv^T \).

4. Let \( u \in \mathbb{C}^n \), \( v \in \mathbb{C}^n \), and let \( E = I - uv^T \in \mathbb{C}^{n \times n} \).

   (a) Show that if \( v^T u = 1 \), then there is a nonzero \( x \in \mathbb{C}^n \) such that \( Ex = 0 \). This implies that \( E \) is singular.

   (b) Show that if \( v^T u \neq 1 \), then \( E \) is nonsingular, and \( E^{-1} = I + \alpha uv^T \), where \( \alpha = 1/(1 - v^T u) \).