Math 520
Assignment 4g due October 12, 2001

6. Let $A \in \mathbb{C}^{m \times n}$. Prove that the pivotal columns of $A$ form a basis for the column space of $A$. This shows that the rank of $A$ is the same as the dimension of the column space. (Don’t wave your hands. Don’t quote the results from page 50 of the text. Do make use of the fact that there is a nonsingular $P \in \mathbb{C}^{m \times m}$ such that $A = PE_A$.)

7. Let $A \in \mathbb{C}^{m \times n}$. Show that if $\text{rank}(A) = r$, then there exist vectors $u_1, u_2, \ldots u_r \in \mathbb{C}^m$ and $v_1, v_2, \ldots, v_r \in \mathbb{C}^n$ such that

$$A = \sum_{k=1}^{r} u_k v_k^T.$$

This generalizes one of the results from Assignment 1. You may work this by any valid method. I envision an argument that starts with either a basis for the column space or a basis for the row space.

8. Suppose $A \in \mathbb{C}^{m \times n}$ has rank $r$, and

$$A = \sum_{k=1}^{r} u_k v_k^T$$

for some $u_1, \ldots, u_r \in \mathbb{C}^m$ and $v_1, \ldots, v_r \in \mathbb{C}^n$. Prove that $u_1, \ldots, u_r$ must be a basis for $\mathcal{R}(A)$, and $v_1, \ldots, v_r$ must be a basis for $\mathcal{R}(A^T)$. 