Math 546  Assignment 8  due Thursday, November 4

1. Run the MATLAB code in Exercise 7.6.8 of Watkins (2002) with the modifications described in part (a).

   (a) Type `help pcg` or look in MATLAB’s help facility to find out how to make a history of the residuals. Run the code with \( m = 160 \), and make a semilog plot of the norms of the residuals as a function of iteration using MATLAB’s `semilogy` command. Print out the plot and turn it in.

   (b) Let \( z_j = \| r_j \| \), the norm of the \( j \)th residual. Verify that if the residual convergence is linear, say \( z_j \approx C \rho^j \) for some \( j \), then the semilog plot of the residuals will be approximately a straight line.

   (c) What sort of convergence does your plot show? That is, is it linear, faster than linear, or slower than linear?

2. Let \( J(v) = \frac{1}{2}a(v, v) - \langle f, v \rangle_0 \) be a quadratic functional defined on a Sobolev space \( H \), as discussed in class. In particular, \( a(\cdot, \cdot) \) is an inner product on \( H \), so it satisfies \( a(v, v) > 0 \) for all nonzero \( v \in H \). Prove that if \( u \in H \) does not minimize \( J \), then there exists \( w \in H \) such that \( a(u, w) \neq \langle f, w \rangle_0 \).